

What is $2 \times 0 \times 1 + 1$?

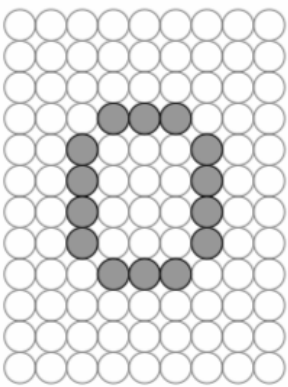
We have $2 \times 0 \times 1 + 1 = 0 + 1 = 1$

How many of the integers 123, 234, 345, 456 and 567 are multiples of 3 ?

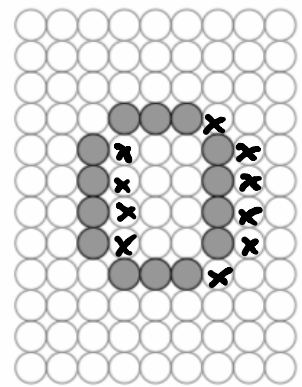
All of them are since the digit sums are 6, 9, 12, 15 and 18 respectively, which are multiples of 3.

Hence the answer is **5**

Letters are shown by lighting cells in a grid, such as the letter 'o' shown. A letter is made bold by also lighting any unlit cell immediately to the right of one in the normal letter. How many cells are lit in a bold 'o' ?



A cross is marked on the additional lit cells. The total number of lit cells is **24**



The world's largest coin had mass 100kg. A standard British £1 coin has mass 10g. What sum of money in £1 coins has the same mass as the record-breaking coin ?

$$\begin{aligned} 1 \text{ kg} &= 10^3 \text{ g} \\ \Rightarrow 100 \text{ kg} &= 10^2 \times 10^3 \text{ g} \\ &= 10^5 \text{ g} \end{aligned}$$

$$= 10^4 \times 10^9$$

So the answer is £10 000

Mother gave each of her children one-twelfth of a chocolate bar. One-third of the bar was left.

How many children does she have?

Suppose she has n children.

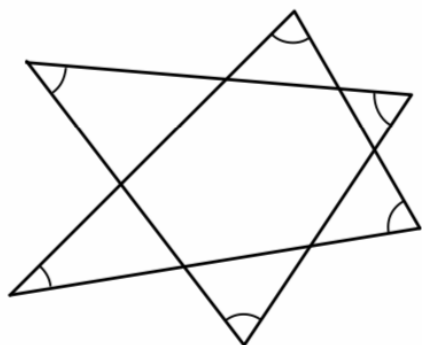
Then
$$1 - n\left(\frac{1}{12}\right) = \frac{1}{3}$$

$$\Rightarrow 1 - \frac{1}{3} = \frac{n}{12}$$

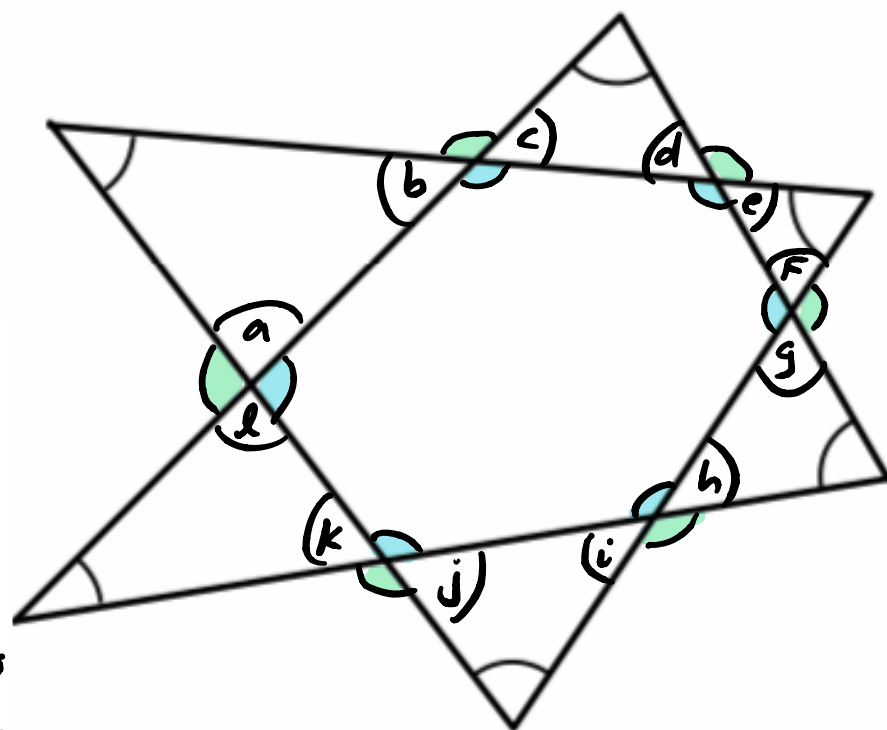
$$\Rightarrow \frac{2}{3} \times 12 = n$$

$$\Rightarrow n = 2 \times 4 = 8$$

What is the sum of the marked angles in the diagram?



Label the angles as shown.



The sum of the blue angles is the sum of the interior angles of a hexagon, which is

$$(6-2) \times 180 = 720$$

By vertically opposite angles, the sum of the green angles is also 720.

The sum of the marked angles is

$$(180-a-b) + (180-c-d) + \dots + (180-k-l)$$

$$= 6 \times 180 - (a + \dots + l)$$

$$= 1080 - (6 \times 360 - 2 \times 720) \quad [\text{angle sum at a point}]$$

$$= 360^\circ$$

Peter Piper picked a peck of pickled peppers.

$$1 \text{ peck} = \frac{1}{4} \text{ bushel}$$

$$1 \text{ bushel} = \frac{1}{9} \text{ barrel}$$

How many more pecks must Peter Piper pick to fill a barrel?

$$\text{We have } 1 \text{ bushel} = 4 \text{ pecks}$$

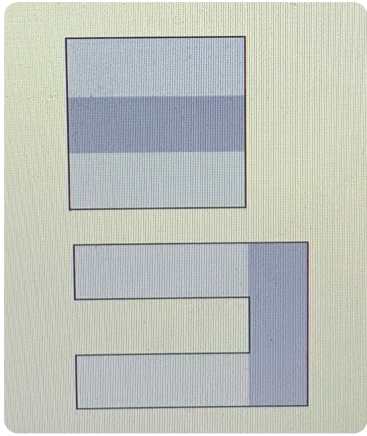
$$1 \text{ barrel} = 9 \text{ bushels}$$

$$= 36 \text{ pecks}$$

Since he already picked 1, he needs **35** more

A square is divided into three congruent rectangles. The middle rectangle is removed and replaced on the side of the original square to form an octagon as shown. What is the ratio of the length of the perimeter of the

Square to the length of the perimeter of the octagon?



Suppose the square has side length $3x$. Then the rectangle has long side $3x$ and short side x .

The perimeter of the square is

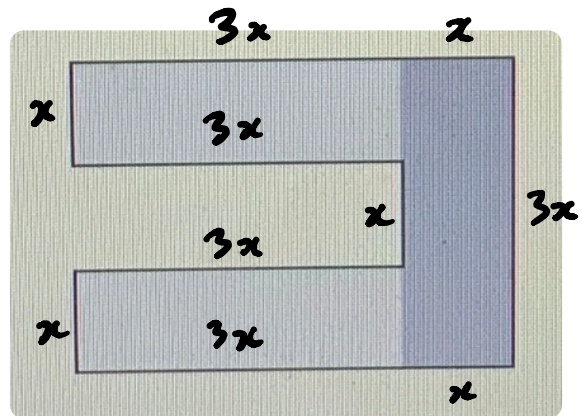
$$4 \times 3x = 12x$$

The perimeter of the octagon is

$$5x + 5(3x) = 20x$$

So the ratio of the perimeters is

$$\frac{12x}{20x} = \frac{3}{5}$$



What is the smallest possible difference between two different nine-digit integers, each of which includes all of the digits 1 to 9?

To minimise the difference, leave all digits except the units and tens column the same.

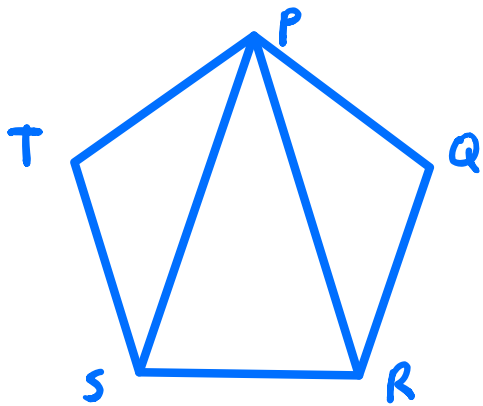
When choosing digits for those columns, minimise the difference between them: so in one number the units digit is n and the tens digit is $n+1$, while in the other number the tens digit is n and the units digit is $n+1$.

The smallest possible difference is therefore

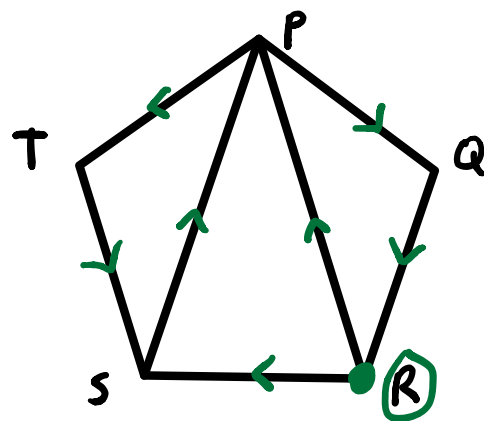
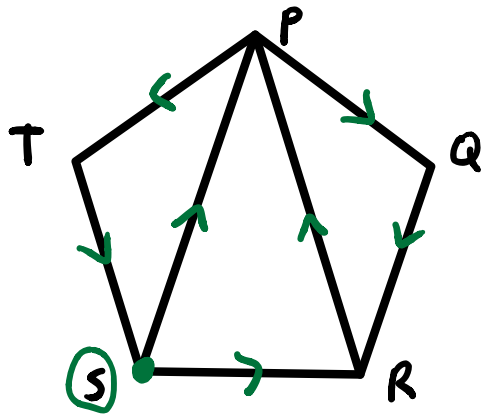
$$\begin{aligned}
 & 10(n+1) + n - [10n + (n+1)] \\
 &= 10n + 10 + n - 10n - n - 1 \\
 &= 10 - 1 \\
 &= 9
 \end{aligned}$$

Example: 987654321 - 987654312

You want to draw this shape without taking your pen off the paper and without going over any line more than once. Where should you start?



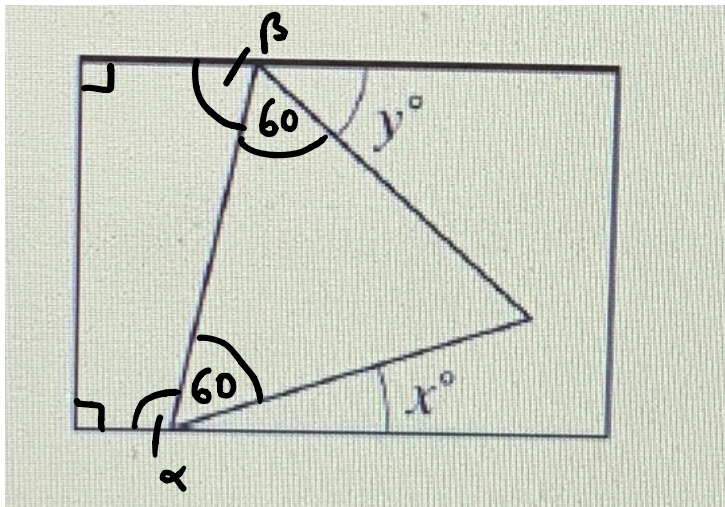
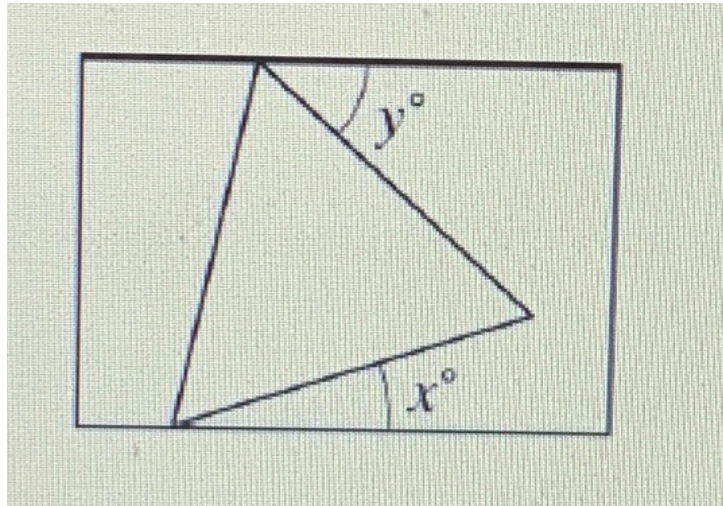
- A) only at T or Q
- B) only at P
- C) only at S or R
- D) at any point
- E) the task is impossible



So it is possible to start at S or R.

It is impossible to start at any other point. If you did, then you would need to enter both S and R twice and leave only once, which is impossible if you don't use one as a start point. So the answer is C

The diagram shows an equilateral triangle inside a rectangle. What is the value of $x+y$?



We have

$$\begin{aligned}\alpha &= 180 - 60 - x \\ &= 120 - x\end{aligned}$$

$$\begin{aligned}\beta &= 180 - 60 - y \\ &= 120 - y\end{aligned}$$

by supplementary angles

By angle sum of a quadrilateral:

$$\alpha + \beta = 180$$

$$\Rightarrow 240 - (x+y) = 180$$

$$\Rightarrow x+y = 240 - 180 = 60^\circ$$

If $\triangle + \triangle = \square$

$\square + \triangle = \bigcirc$

$\diamond = \bigcirc + \square + \triangle$

how many $\triangle$'s are equal to $\diamond$?

We have $\diamond = \bigcirc + \square + \triangle$

$$= (\square + \triangle) + (\triangle + \triangle) + \triangle$$

$$= \triangle + \triangle + \triangle + \triangle + \triangle + \triangle$$

So the answer is 6

What is the mean of $\frac{2}{3}$ and $\frac{4}{9}$?

$$\begin{aligned} \text{Mean} &= \frac{1}{2} \left(\frac{2}{3} + \frac{4}{9} \right) \\ &= \frac{1}{2} \left(\frac{6}{9} + \frac{4}{9} \right) \\ &= \frac{5}{9} \end{aligned}$$

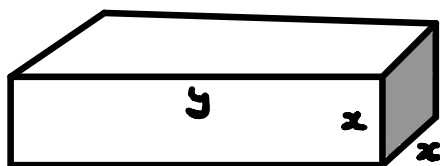
The diagram shows a cuboid in which the area of the shaded face is one-quarter of the area of each of the two visible unshaded faces.

The total surface area of the cuboid is 72cm^2 .

What is the area of one of the visible unshaded faces of the cuboid?



Suppose that the shaded face, a square, has side length x and the other side of the unshaded face has length y



The area of a shaded Face is x^2

The area of an unshaded Face is xy

So

$$x^2 = \frac{1}{4} xy$$

$$\Rightarrow y = 4x$$

The total surface area of the cuboid is

$$4(4x)x + 2x^2 = 16x^2 + 2x^2 = 18x^2$$

$$\text{So } 18x^2 = 72 \Rightarrow x^2 = 4$$

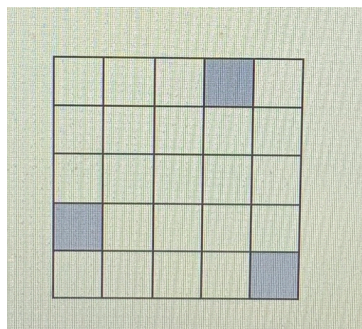
$$\Rightarrow x = 2$$

$$\Rightarrow y = 8$$

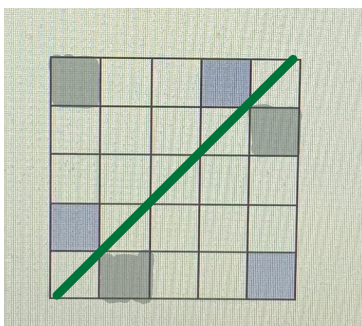
and hence the area of an unshaded surface is

$$xy = 2 \times 8 = 16 \text{ cm}^2$$

What is the smallest number of additional squares which must be shaded so that this figure has at least one line of symmetry and rotational symmetry of order 2?



To create rotational symmetry we must shade the indicated squares. But this also creates a line of symmetry (green) so the answer is 3



An election candidate wins if they receive more votes than any other candidate. There are 83 votes total. What is the smallest number of votes the winner could receive in a 4-person election?

To minimise the number of votes, the winning margin should be as small as possible.

We have

$$83 = 4 \times 20 + 3$$

$$= 20 + 20 + 21 + 22$$

Since $83 \div 4 = 20.75$, at least one candidate must receive 21 votes or more. If they received 21, the other 3 candidates must have a total of 62 between them, so they can't all receive 20 votes or less.

It is possible for the winner to receive 22 votes as shown above, so the answer is **22**

The longest tennis match in history lasted 11 hours 5 minutes. The Fifth set took 8 hours 11 minutes.

Approximately what fraction of the whole match was taken up by the Fifth set?

A) $\frac{1}{5}$ B) $\frac{2}{5}$ C) $\frac{3}{5}$ D) $\frac{3}{4}$ E) $\frac{9}{10}$

$$\text{Whole match time} = 11 \times 60 + 5 \text{ min}$$

$$= 665 \text{ min}$$

$$\text{Fifth set} = 8 \times 60 + 11 \text{ min}$$

$$= 491 \text{ min}$$

So the approximate proportion taken by the Fifth set is

$$\frac{491}{665} \approx \frac{500}{666} = \frac{3}{4}$$

Peri leaves on Monday to visit Granny, 90m away. Except For rest days, Peri travels 1m every day at a constant rate and without pause. He rests for the whole day every 10th day. On which day of the week does he arrive?

Every 10 days he travels 9m. After 90 days he has travelled 81m. After a further 9 days he arrives, so it takes him 99 days.

We have

$$99 = 14 \times 7 + 1$$

So he arrives on a **Tuesday** (14 weeks + 1 day)

A list is made of every digit that is the units digit of at least one prime number. How many of the Following numbers are in the list?

1 2 3 4 5

1 can - eg 11

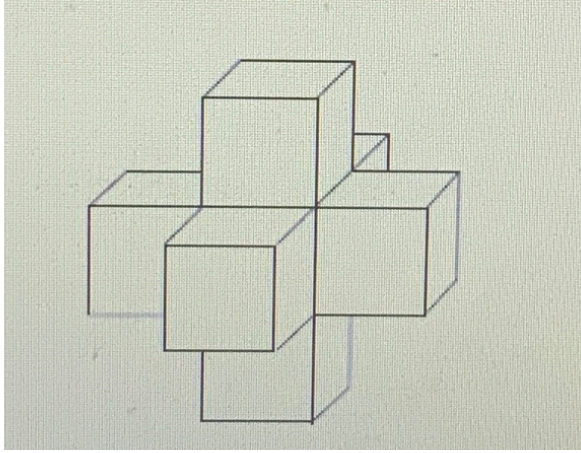
2 can - eg 2

3 can - eg 3

5 can - eg 4

4 cannot since every number ending in 4 is a multiple of 2. Hence the answer is **4**

One cube has each of its faces covered by one face of an identical cube, making a solid as shown. The volume of the solid is 875 cm^3 . What is the surface area?



The solid is comprised of 7 identical cubes, so the volume of a single cube is

$$875 \div 7 = 125$$

and so it has side length

$$\sqrt[3]{125} = 5$$

The surface area is 6 times the surface area of a single cube minus the surface area of a single cube:

$$5 \times (5 \times 5 \times 6) = 750 \text{ cm}^2$$

Grill takes a train at 9:00. The train travels for the first 27 km at 96 km/h. It stops for 3 min before travelling the final 29 km at 96 km/h. At what time does it arrive?

It travels $27 + 29 = 56 \text{ km}$ at 96 km/h, which takes

$$\text{time} = \frac{\text{speed}}{\text{dist}} = \frac{56}{96} = \frac{7}{12} \text{ hours}$$

$$= 7 \times \frac{60}{12} \text{ min}$$

$$= 35 \text{ min}$$

Add the 3 min stop - so it arrives at **9:38**

Evariste and Sophie both bought some stamps. Each stamp Evariste bought cost him £1.10 while Sophie paid 70p for each of hers. Between them they spent exactly £10. How many stamps did they buy in total?

Let e be the number of stamps Evariste buys and s be the number of stamps Sophie buys.

We have:

$$110e + 70s = 1000$$

$$\Rightarrow 11e + 7s = 100$$

Count backwards From 100 by 11's until we reach a multiple of 7:

$$100, 89, 78, 67, 56 = 7 \times 8$$

$\begin{array}{cccc} \curvearrowright & \curvearrowright & \curvearrowright & \curvearrowright \\ -11 & -11 & -11 & -11 \end{array}$

So we have $100 = 11 \times 4 + 7 \times 8$

Hence $e = 4$, $s = 8$ and the total number of stamps they purchased was $4 + 8 = 12$

The points S, T, U lie on the sides of the triangle PQR as shown so that $QS = QU$, $RS = RT$.

$\angle TSU = 40^\circ$. What is $\angle TPU$?

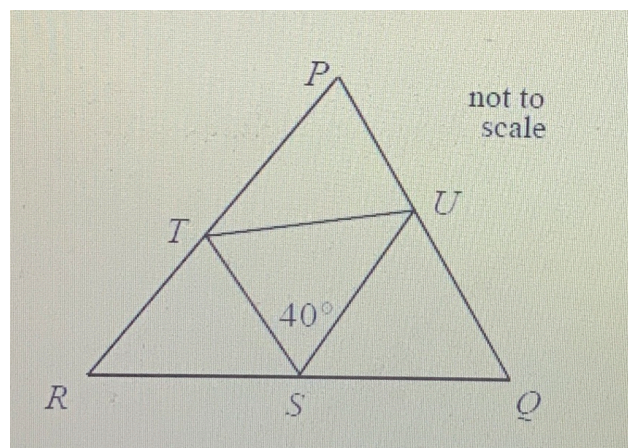
Let $x := \angle RTS$.

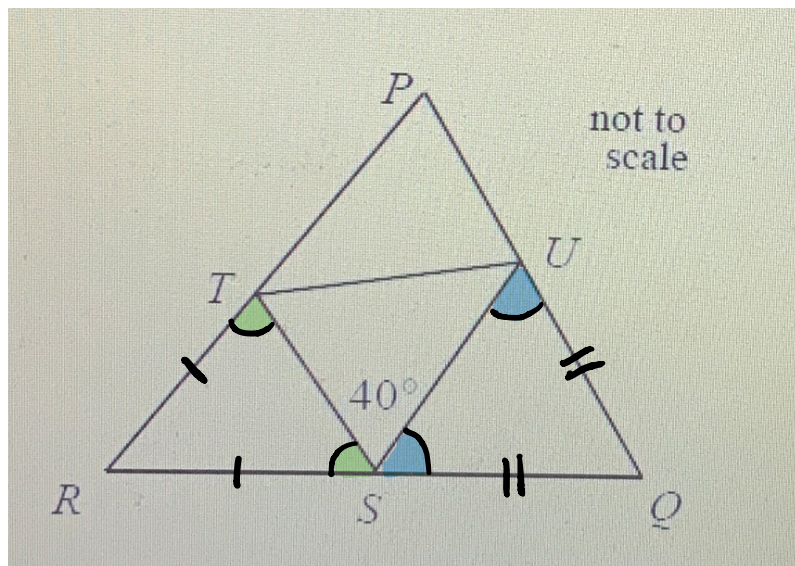
Since $RS = RT$, $\angle RST = x$

By angle sum of a triangle

$$\angle TRS = 180 - 2x$$

By supplementary angles





$$\begin{aligned}\angle USQ &= 180 - 40 - x \\ &= 140 - x\end{aligned}$$

Since $QS = QU$, we have

$$\begin{aligned}\angle QUS &= \angle USQ \\ &= 140 - x\end{aligned}$$

By angle sum of a triangle

$$\begin{aligned}\angle SQU &= 180 - 2(140 - x) \\ &= 2x - 100\end{aligned}$$

Now in $\triangle PQR$ we have

$$\begin{aligned}\angle P &= 180 - \angle R - \angle Q \\ &= 180 - (180 - 2x) - (2x - 100) \\ &= 100^\circ\end{aligned}$$

Two adults and two children wish to cross a river. They make a raft but it will carry only the weight of one adult or 2 children. What is the minimum number of times the raft must cross the river to get all four people to the other side? (It may not cross without at least 1 person on board)

A journey From the other side of the river back to this side with only an adult in the raft is pointless, so all return trips must be made by kids.

1st trip: 2 kids $\rightarrow$

2nd trip: 1 kid $\leftarrow$

3rd trip: 1 adult $\rightarrow$

4th trip: 1 kid $\leftarrow$

Now both kids are back on this side so

5th trip: 2 kids $\rightarrow$

6th trip: 1 kid $\leftarrow$

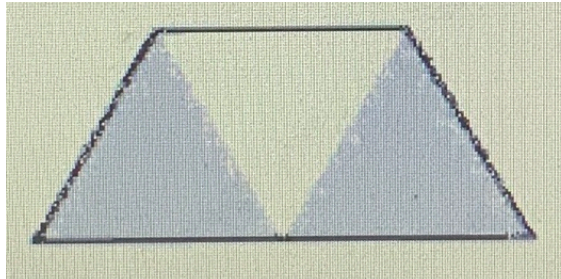
7th trip: 1 adult $\rightarrow$

8th trip: 1 kid $\leftarrow$

9th trip: 2 kids $\rightarrow$

So the answer is 9

The diagram shows a trapezium made From three equilateral triangles. Three copies of the trapezium are placed together without gaps or overlaps and so that only complete edges coincide, to form a polygon with N sides. How many different values of N are possible?





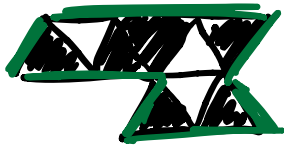
8 sides ✓



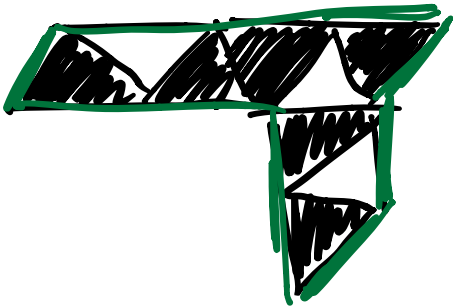
4 sides ✓



7 sides ✓



7 sides
x



7 sides
x

4 sides on 3 pieces $\rightarrow$ 12 total

Combining them in 2 places

each join takes either 1 pair (= 2 sides)

or 6 sides (2 sets of 3)